Risk and Asset Prices in a Monetary Model*

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Abstract

In this paper, we develop a monetary model that is consistent with empirical evidence on how a central bank's actions affect the macroeconomy and financial markets. In our setting, the monetary transmission mechanism relies on two ingredients: market segmentation and liquidity frictions. An expansionary monetary policy provides liquidity to agents who participate in financial markets and insures them against capital income risk. This leads to higher investment, higher aggregate consumption in the future, and a higher share of non-wage income in the economy, as well as an increase in stock market valuations, lower expected equity returns and a lower return volatility in the financial markets.

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1 Introduction

In this paper, we develop a monetary model that is consistent with empirical evidence on how the central bank's actions affect the macroeconomy and financial markets. The monetary transmission mechanism in our setting relies on two ingredients: market segmentation and liquidity frictions. An expansionary monetary policy provides liquidity to agents who participate in financial markets and insures them against capital income risk. This leads to higher investment, higher aggregate consumption in the future, and a higher share of non-wage income in the economy, as well as an increase in stock market valuations, lower expected equity returns and return volatility in the financial markets.

An extensive empirical literature documents the effects of monetary surprises on aggregate quantities, inflation, and financial market variables. Independent of how these surprises are identified, there is a consensus that during an expansionary monetary episode, aggregate output and consumption are higher after a lag, inflation barely moves, capital income shares are higher, nominal rates and real rates are low at several maturities, asset valuations are high, expected returns and variance of returns are low. These patterns constitute a set of informative moments for the transmission channel that underlie monetary models.

Conventional New Keynesian (NK) models (based on Woodford (2011) or Gali (2015)) that are widely used in the literature are inconsistent with several of the patterns uncovered in the empirical literature. In these models, the effects of monetary policy on the macroeconomy and financial markets are driven by the ability of nominal rate surprises to affect real rates and spur aggregate demand. Effects on equity premia and variance of returns are largely negligible or absent in the often-studied approximate equilibria that rely first- or second-order expansions. Consider a couple of examples to illustrate the nature of these inconsistencies. Canzoneri et al. (2007) observe that it is difficult for conventional models to reconcile a negative correlation between real rates and aggregate consumption growth that is found in the data. Bernanke and Kuttner (2005) conduct a decomposition of the identified stock valuation effects and show that time-varying equity premia drive these effects; while conventional models would conclude that they are almost exclusively driven by movements in risk-free rates. Similar observations are made by Hanson and Stein (2015) who look at long-maturity real rates, and Bekaert et al. (2013a) who study movements in VIX. The evidence taken together casts a serious doubt on the central property of these models, that is, how central bank policy affects macro outcomes, risk exposures, and risk premia.

We propose a model of monetary transmission that overcomes the deficiencies in conventional models. Our setup has a liquidity role for money which is modeled as a cash-in-advance constraint on households. In addition, we allow for segmentation in financial markets. In particular, a subset of agents face capital income risk and trade in asset markets, and participate in open market operations with the monetary authority. The nonparticipating households rely on nominal wage income. In this setup, we study the effects of a surprise increase in nominal money balances.

Expansionary monetary policy is implemented by the central bank buying bonds and issuing money. In our model, it raises consumption and investment spending by participating households and consequently increases output and inflation. There is also a redistributive effect of monetary expansion – it lowers real wages and transfers resources from the nonparticipating households towards the participating households. This transfers hedges the participating households against the background capital income risk and increases their risk-taking propensity. General equilibrium forces imply lower risk-free rates, lower equity premia, and lower returns volatility. Thus the model can reconcile evidence from macro and the asset markets in a simple and transparent way.

Our modeling of liquidity frictions and segmented markets builds on Alvarez et al. (2001) and Alvarez et al. (2009). Relative to those papers, we add production, time-varying markups, and study a broader set of implications on macro and financial variables. In our setup, monetary policy works through a redistribution channel that is similar to Doepke and Schneider (2006) and several papers in the growing Heterogeneous agent New Keynesian literature, for example, Auclert (2017), Kaplan et al. (2016).

A related strand of literature builds market segmentation in conventional NK models such as Bernanke et al. (1999), Brunnermeier and Sannikov (2014), Kekre and Lenel (2019) by modeling agents that differ in risk preferences or in their access to risky investment opportunities. In those setups, monetary transmission works through the conventional aggregate demand channel, and its effects are amplified by movements in wealth shares across agents. We next review the empirical evidence on the effects of monetary surprises and discuss in more details why existing models are inconsistent with the evidence.

2 Some Facts for Monetary Models

In this section, we review the evidence on the causal effects of monetary shocks on macro and financial variables. Monetary shocks have been identified in several ways such as Cholesky ordering (Christiano et al. (1999)), narrative approach (Romer and Romer

(2004)), sign restrictions (Uhlig (2005)), high-frequency movements in interest rate futures (Kuttner (2001)). The patterns we summarize below are robust to identification schemes. For concreteness, consider the effects of a surprise monetary expansion. Facts 1 - 2 describe responses to the first and second moments of macro aggregates. Facts 3 -7 discuss the effects on asset markets including short and long-term bonds, equities, and VIX.

Fact 1: A positive and delayed response of aggregate output and a small response of inflation The fact that monetary shocks affect real quantities and agregate inflation is probably the most comprehensively documented pattern. See Ramey (2016) for an extensive survey. The estimates for the peak response of output vary between 100 - 200 basis points with a lag of usually 10 quarters or so. Thus, the expected growth rate of output is positive after a monetary expansion. The effects on aggregate consumption are similar but smaller in magnitude. In addition, a slow, attenuated, and sometimes negative response (also referred to as the "price puzzle") of the price level is a common finding.

Fact 2: A negative correlation of short-term real rates and aggregate consumption growth An expansionary monetary stance is often identified as lowering of the feds funds rate. Building on Fact 1, Canzoneri et al. (2007) points out that real short-term risk-free rates (defined short-term nominal rates minus expected inflation) co-move negatively expected growth rate of consumption. As we explain later, this poses a limitation for several existing monetary models.

Fact 3: A large and positive response for non-wage income as compared to wage income Disaggregating the increase in GDP by different components of income, Coibion et al. (2017) find a substantial increase in the share of non-wage components especially income from business and transfers and a statistically small effect on nominal wages.

Fact 4: A positive response of stock valuations Bernanke and Kuttner (2005), Rigobon and Sack (2003) and others document that a large increase in broad stock indexes after a monetary expansion. The point estimates range between 100 and 200 basis points for a 25 basis point reduction in the policy rate.

Fact 5: A negative response of expected returns and equity premia Bernanke and Kuttner (2005) also use a Campbell and Shiller (1988) methodology to decompose the increase in stock valuations into news about higher cash flows, news about lower future risk-free rates and a residual component due to lower current and future equity premia. They find little contribution from the risk-free rates and conclude that that policy's impact on equity prices comes in through its effect on lower expected future excess equity returns and partly due to higher cash flows.

Fact 6: A negative response of conditional and risk-adjusted volatility of returns Bekaert et al. (2013b) document that VIX, which is the stock market option-based implied volatility shows a strong positive response after a monetary expansion. They also decomposed the VIX into two components, a proxy for risk aversion and expected stock market volatility to find that a lax monetary policy decreases both risk aversion and uncertainty, with the former effect being dominant.

Fact 7: A negative response for long-term real rates Hanson and Stein (2015) study the effect of monetary policy on real long-term risk-free rates. Using data on interest rates forwards in the US and UK, they document that a 100 basis point increase in the two-year nominal yield on a Federal Open Markets Committee announcement day is associated with a 42 basis point increase in the ten-year forward real rate.

2.1 Implications of Conventional Models

Monetary models used in the literature come in various varieties and several are built on a common "New Keynesian" (NK) core. This core includes monopolistically competitive intermediate good producers that face nominal frictions in price setting, households that save and supply labor on competitive spot markets, and monetary authority that sets the nominal rate using a Taylor rule. See for example, the textbook treatment in Woodford (2011) or Gali (2015). The transmission mechanism of monetary shocks relies on nominal frictions and through that, the ability of monetary shocks to affect aggregate demand. An unanticipated decrease in short-term nominal rate lowers the real rate due to rigidity in price setting. Households facing this low real rate, desire higher current consumption. With nominal frictions, output is demand determined, and firms respond by hiring more workers and producing higher output and partly increasing prices. This results in higher aggregate output, consumption, and inflation. The effects are short-lived and decay with the persistence of the monetary shock.

The transmission mechanism has several implications. Most importantly, the aggregate demand channel works through an aggregate Euler equation that imposes to the first order a positive co-movement between consumption growth and real rate. As emphasized by Canzoneri et al. (2007), this is inconsistent with Fact 1 and Fact 2. In the basic version of the model with only nominal frictions on price setting, real wages are higher in monetary expansion, and firm profits or markups are lower. This is inconsistent with Fact 3. The inflation response is positive and quantitatively large and inconsistent with Fact 1.

The effects of monetary shocks on second-moments are negligible or absent in most implementations that rely on first-order or second-order approximations. This implication, in turn, uncovers several other failures of conventional models. Firstly, risk premia are absent (or constant). Although asset valuations are higher, they are entirely due to lower real rates which is grossly at odds with Bernanke and Kuttner (2005) evidence in Fact 5. Lack of movements in higher-order terms naturally means that these models are incapable of hitting VIX related Fact 6 documented in Bekaert et al. (2013b). In these models, the expectation hypothesis holds and given the transitory nature of monetary shocks, expected long-term real rates are equal to their steady-state value. As emphasized in Hanson and Stein (2015), this is inconsistent with Fact 7.

Admittedly, the conclusions in the previous paragraphs are drawn from the most basic version of the NK model. Some implementations such as Christiano et al. (1999) add habits persistence to get Fact 1, some versions such as Erceg et al. (2000) add sticky wages to get Fact 3. A few depart from the representative agent, such as Bernanke et al. (1999) and Kekre and Lenel (2019). In Bernanke et al. (1999), there are two types of agents: risk-averse savers and risk-neutral investors who financial frictions. The model relies on the aggregate demand channel and large investment adjustment costs to generate higher asset valuations in monetary expansions. The demand-driven expansions increase the wealth share of the marginal investors and relaxes their financial frictions amplifying the effects of the monetary shocks. In a mechanism similar to Bernanke et al. (1999), Kekre and Lenel (2019) use heterogeneous risk aversion to generate effects on risk premia. Like the conventional models, monetary shocks affect aggregate demand and inflation, and like in Bernanke et al. (1999), shuffles wealth across agents due to the presence of inter-agent nominal claims. Movements in wealth shares across agents with different risk aversion then affects the overall risk tolerance, and as long as these movements are induced by monetary shocks, they show up as movements in risk premia following monetary shocks.

In contrast to all these papers, we propose a mechanism based on segmented markets that hits all the facts mentioned in section 2. We build on the Alvarez et al. (2001) and show that monetary injections through open market operations provide liquidity services and insurance to the participants in the open market operations who are the marginal investors for all financial claims. Our mechanism does not rely on sticky prices or aggregate demand channel of monetary shocks. In section 3, we lay out the environment. In section 4, we use a simple version of the model to analytically why the model generates Facts 1 through 7, and then in section 5, we study a calibrated version.

3 Setup

The economy is populated by two types of households, workers and capitalists with mass λ and $1-\lambda$ respectively. The types denote (i) factor endowments, i.e., workers supply labor, and capitalists supply capital and own firms that employ factors and produce goods, and (ii) access to asset markets which we elaborate later. We use superscript W to index workers and superscript C to index capitalists. There is combined monetary and fiscal authority that decides on bonds, money supply, and taxes. Monetary policy, as in practice, is conducted using open market operations, where nominal bonds are exchanged for money.

Workers We start with the worker households. Workers are each endowed with one unit of available time, segmented from bond and equity markets, and face a cash-in-advance constraint that requires them to finance the current period spending using after-tax nominal money balances. Their maximization problem is formulated as follows:

$$\max_{C_t^W, M_t^W, N_t^W} \mathbb{E}_0 \sum \beta^t U^W \left(C_t^W, N_t^W\right)$$

subject to the nominal budget constraint

$$P_t C_t^W + M_t^W \leq W_t N_t^W - T_t^W + M_{t-1}^W. \tag{1}$$

Taxes T_t^W are lump-sum and M_{t-1}^W are the money balances carried over from $t-1$. The workers face a cash-in-advance constraint

$$P_t C_t^W \leq M_{t-1}^W - T_t^W. \tag{2}$$

Capitalists The capitalists are endowed with capital. They can trade bonds, equities, and participate in open market operations with the monetary-fiscal authority. As in Alvarez et al. (2001), assets markets open before goods market, i.e., at beginning of time t, capitalists enter the asset markets with a portfolio of money and bonds M_{t-1}^C, B_{t-1} and buy or sell new bonds at a price Q_t. The money balances after the open market operations are given by $M_{t-1}^C + B_{t-1} - Q_t B_t$, and are available for spending in the goods markets in time t. The capitalists' maximization problem is given by

$$\max_{C_t^C, K_t, B_t} \mathbb{E}_0 \sum \beta^t U^C(C_t^C)$$

subject to flow budget constraints

$$P_t \left(C_t^C + I_t\right) + M_t^C + Q_t B_t \leq P_t \left(Pr_t + R_t^k K_{t-1}\right) - T_t^C + M_{t-1}^C + B_{t-1}, \tag{3}$$

$$K_t = (1-\delta) K_{t-1} + I_t.$$

The capitalists' income consists of profits from the intermediate goods firms, Pr_t, and rental income to the capital they supply, $R_t^k K_{t-1}$. As in the case with workers, they pay a lump-sum tax T_t^C and face a cash-in-advance constraint

$$P_t(C_t^C + I_t) \leq M_{t-1}^C + B_{t-1} - Q_t B_t - T_t^C. \tag{4}$$

In our formulation the cash-in-advance constraints apply to both investment and consumption.[1]

Production Next, we describe the supply side of the economy. Production takes place in two layers – monopolistically competitive firms (owned by capitalists) produce intermediate goods varieties and these are aggregated by a competitive final goods producers.

Let i denote a variety or equivalently a intermediate good firm. The final goods producers use constant elasticity of substitution (CES) technology

$$Y_t = \left[\int y_t(i)^{\frac{\varepsilon_t - 1}{\varepsilon_t}} di\right]^{\frac{\varepsilon_t}{\varepsilon_t - 1}},$$

[1] Our assumption that investments are cash goods follows Stockman (1981). Alternatively, Cooley and Hansen(Cooley and Hansen (1989)) model investment as credit goods. Our results are not sensitive to either way of modeling the cash-in-advance constraint.

and their cost minimization yields a downward sloping demand curve

$$y_t(i) = \left(\frac{p_t(i)}{P_t}\right)^{-\varepsilon_t} Y_t$$

for the each variety i with the aggregate price index P_t defined by

$$P_t \equiv \left[\int p_t(i)^{1-\varepsilon_t}\right]^{\frac{1}{1-\varepsilon_t}}.$$

We allow time variation in the the elasticity parameter $\{\varepsilon_t\}$. These are exogenous disturbances in the profits and the wealth of the capitalists.

The intermediate firms use a constant returns to scale technology to produce

$$y_t(i) = Ak_{t-1}^{\alpha}(i)n_t^{1-\alpha}(i)$$

Given prices $\{P_t, W_t, R_t^k\}$ and demand from final goods producers, the intermediate goods firm choose capital and labor to maximize period-by-period profits

$$pr_t(i) \equiv \max_{\{p_t(i),k_{t-1}(i),n_t(i)\}} \left(\frac{p_t(i)}{P_t}\right)^{1-\varepsilon_t} Y_t - \left(\frac{W_t}{P_t}\right) n_t(i) - R_t^k k_{t-1}(i).$$

All the non-wage income, i.e., profits (markups) and rents on capital are remitted to the capitalists. Aggregate production is exhausted by consumption and investment on the product side, and wage and returns to operating capital/owning firms on the income side.

$$Y_t = \lambda C_t^C + (1-\lambda)C_t^W + \lambda I_t = \lambda\left(R_t^k K_t + Pr_t\right) + (1-\lambda)N_t\frac{W_t}{P_t}.$$

Government Finally, we describe the monetary-fiscal policies. Let M_t and T_t be aggregate nominal balances and nominal transfers

$$(1-\lambda)M_t^W + \lambda M_t^C = M_t,$$

$$(1-\lambda)T_t^W + \lambda T_t^C = T_t.$$

The stochastic processes for $\{M_t, T_t^W, T_t^C\}$ are exogenously specified and satisfy a budget constraint

$$M_{t-1} + \lambda B_{t-1} = \lambda Q_t B_t + M_t + T_t. \tag{5}$$

Equilibrium The concept of an equilibrium is standard. Given initial conditions $\{M_{-1}, B_{-1}\}$ and exogenous processes $\{B_t, M_t, T_t^W, T_t^C, \varepsilon_t\}$, a competitive equilibrium constitutes of prices $\{P_t, W_t, R_t^k\}$, allocations $\{C_t^W, C_t^C, N_t^W, M_t^W, M_t^C, K_t\}$ such that workers, capitalists, and firms optimize and goods, labor, and asset markets clear and the government budget constraint is satisfied.

4 Analytically Tractable Case

In this section, we study a special case of the model that is analytically tractable and conveys the intuition for why our mechanism can hit all the facts mentioned in section 2. We construct the analytically tractable version in two steps. We first study a version without investment and taxes/transfers to show that it can generate Facts 3-7, and in the next step, we discuss how adding investment and fiscal policy can generate the remaining two, i.e., Facts 1 and 2.

We impose the following functional forms for utility functions:

$$U^W(C_t^W, N_t^W) = \ln C_t^W - \frac{\beta}{1+\varphi}\left(N_t^W\right)^{1+\varphi},$$

$$U_t^C(C_t^C) = \frac{\left(C_t^C\right)^{1-\gamma}}{1-\gamma}.$$

Let $\Phi_t \equiv \frac{\varepsilon_t - 1}{\varepsilon_t}$, the stochastic process for desired markups is given by

$$\ln \Phi_t = \bar{\phi} + \sigma \epsilon_t$$

We also impose enough assumptions on the primitives such that the cash-in-advance constraint binds for all agents. We then study the following thought experment. Starting from a stedy state a given money stock (and no inflation), consider an unaticipated reduction in the nominal rates with the path

$$\log Q_{t+j} - \log \beta = \rho^j \left[\log Q_t - \log \beta\right] \tag{6}$$

This reduction in nominal rates is attained by buying back goverment bonds with money. For the first part of the analysis we set capital share to zero, i.e., $\alpha = 0$, and $T_t^W = T_t^C = 0$. In the more general setup later, we will allow for the bond buy-backs to be financed both by taxes and seniorage. We begin by characterizing the equilibrium allocation starting with the labor supply.

Proposition 1. *If* $U^W(C_t^W, N_t^W) \equiv \ln C_t^W - \frac{\beta}{1+\varphi}\left(N_t^W\right)^{1+\varphi}$ *and* $T_t^W = 0$*, then* $N_t = 1$ *for all* t

Proof. The first order condition with respect to N_t^W is

$$U_N\left(N_t^W\right) + \beta \mathbb{E}_t \frac{W_t}{P_{t+1}} U_c\left(C_{t+1}^W\right) = 0$$
$$\left(N_t^W\right)^{\varphi} = \mathbb{E}_t \frac{W_t}{P_{t+1} C_{t+1}^W}$$

From the budget constraint of the worker $P_{t+1}C_{t+1}^W = W_t N_t^W$ and substituting in the previous equation, we can verify that $N_t^W = 1$. □

Proposition 1 states that labor supply, and hence total output is fixed. Our next result characterizes the distribution of consumption across the two types of agents.

Proposition 2. *Let* X_t *be the growth rate of money supply. The consumption share of capitalist is*

$$\frac{C_t^C}{Y_t} = \frac{1}{\lambda}\left(1 - \frac{\Phi_{t-1}}{X_T}\right)$$

where $X_t = \chi(Q_t)$ *with* $\chi' > 0$ *as long as* ρ *is not too large.*

Proof. Combine the cash-in-advance constraints of workers and capitalists, equations (2), and (4) with the budget constraint of the government in equation (5) to obtain the quantity of money equation

$$P_t Y_t = M_t$$

The cash-in-advance constraint of the capitalist (4) can be expressed as

$$\lambda P_{t+1} C_{t+1}^C = \left(P_t Y_t - (1-\lambda) W_t N_t^W\right) + \left(X_{t+1} - 1\right) M_t$$

Using $N_t^W = 1$ from Proposition 1 and $\frac{W_t}{P_t} = \frac{1}{\Phi_t}$ obtain

$$C_{t+1}^C = \frac{\frac{1-\lambda}{\lambda}\left(1 - \Phi_t\right) + \frac{1}{\lambda}\left(X_{t+1} - 1\right)\frac{M_t}{P_t}}{P_{t+1}/P_t}$$

Finally use $Y_t = N_t^W = 1 - \lambda$ and substitute $P_t = \frac{M_t}{1-\lambda}$ and define $M_t = X_t M_{t-1}$ to get

$$\frac{C_t^C}{Y_t} = \frac{1}{\lambda}\left(1 - \frac{\Phi_{t-1}}{X_t}\right).$$

The relationship between X_t and Q_t is given by

$$Q_t \left(\frac{X_t - \Phi_{t-1}}{X_t}\right)^{-\gamma} = \beta \mathbb{E}_t \left(\frac{X_{t+1} - \Phi_t}{X_{t+1}}\right)^{-\gamma} \left(\frac{1}{X_{t+1}}\right)$$

Integrating out both: Φ_t, Φ_{t+1} and denoting it gives a relationship between X_t, X_{t+1} and Q_t. Write that as

$$G\left(X_t, X_{t+1}, Q_t\right) = 0 \tag{7}$$

A first-order approximation to (7) with respect to $x_t = \log X_t$ □

$$\log Q_t - \log \beta + (1-\gamma)\, \hat{x}_{t+1} - \gamma \left(\frac{\mathbb{E}\left(1-\Phi_{t-1}\right)^{-\gamma-1}}{\mathbb{E}\left(1-\Phi_{t-1}\right)^{-\gamma}}\right) \hat{x}_t = -\gamma \hat{x}_t - \gamma \left(\frac{\mathbb{E}\left(1-\Phi_t\right)^{-\gamma-1}}{\mathbb{E}\left(1-\Phi_t\right)^{-\gamma}}\right) \hat{x}_{t+1} + \mathcal{O}(\hat{x}^2)$$

Lets denote $\left(\frac{\mathbb{E}(1-\Phi_t)^{--1}}{\mathbb{E}(1-\Phi_t)^{-\gamma}}\right)$ be constant c. Our assumptions imply that $c > 1$ for $\varepsilon > 1$. Now rewrite the previous expression to get

$$x_t = \sum_{j=0}^{\infty} \left[\frac{1+\gamma\,(c-1)}{\gamma\,(c-1)}\right]^j \left(\frac{\log Q_{t+j} - \log \beta}{\gamma\,(c-1)}\right)$$

substitute for Q_{t+j} from (6) we obtain

$$x_t = \left(\frac{[\log Q_t - \log \beta]}{\gamma\,(c-1)}\right) \sum_{j=0}^{\infty} \left[\left(\frac{1+\gamma\,(c-1)}{\gamma\,(c-1)}\right)\rho\right]^j.$$

Therefore, as long as

$$\left(\frac{1+\gamma\,(c-1)}{\gamma\,(c-1)}\right)\rho < 1$$

x_t is increasing in $\log Q_t$.

We use Proposition 1 to derive some properties of the pricing kernel and verify the asset pricing implications of our model. Since the capitalist is the marginal agent for all financial assets, the stochastic discount factor for pricing nominal claims that pay in period t is

$$S_{0,t} = \beta \left(\frac{C_t^C}{C_0^C}\right)^{-\gamma} \frac{P_0}{P_t} = \beta \left(\frac{C_t^C}{C_0^C}\right)^{-\gamma} \frac{1}{X_t}$$

Let $c_t^C = \log C_t^C$ and $x_t = \log X_t$. Using Proposition 2, we see that

$$\begin{aligned} c_{t+1}^C &= \ln\left(\frac{1-\lambda}{\lambda}\right) + \ln\left(1 - \exp\left(\bar{\phi} + \sigma\epsilon_t - \rho^{t+1}x_0\right)\right) \\ &= \frac{1-\lambda}{\lambda}\left(1 - \frac{\bar{\Phi}}{X_{t+1}}\right) + \sigma\frac{\exp\left(\bar{\phi} - \rho^{t+1}x_0\right)}{1 - \exp\left(\bar{\phi} - \rho^{t+1}x_0\right)}\epsilon_t \\ &\quad + \frac{\sigma^2}{2}\frac{\exp\left(\bar{\phi} - \rho^{t+1}x_0\right)}{\left[1 - \exp\left(\bar{\phi} - \rho^{t+1}x_0\right)\right]^2}\epsilon_t^2 \\ &\quad + O\left(\sigma^3\right) \end{aligned}$$

therefore

$$var_{t-1}\left(c_{t+1}^C\right) = \sigma^2\frac{\exp\left(\bar{\phi} - \rho^{t+1}x_0\right)}{1 - \exp\left(\bar{\phi} - \rho^{t+1}x_0\right)}var\left(\epsilon\right) + O\left(\sigma^3\right). \tag{8}$$

Expression (8) shows that an expansionary monetary shock lowers the conditional volatility of capitalist's consumption. The intuition is straightforward. An expansionary monetary shock injects more liquidity to capitalists. They demand more goods and put upward pressure on the price level while lowering real wages. Equation (8) shows that a expansionary monetary shock lowers the conditional volatility of capitalist's consumption. As discussed before, aggregate labor is fixed and hence there is no change in the aggregate output. However, there is a redistribution of resources from the workers in the favor of the capitalists in such expansionary episodes. A larger share of the capitalist's consumption share is driven by the new cash and this provides insurance against the markup fluctuations that they face in the background thereby lowering the conditional volatility of their consumption.

We are now ready to show how our setup reconciles Facts 3 through 8. Non-wage share of total income in our model equals the consumption of the capitalists. Proposition 2 immediately gives us Fact 3 as $\frac{C_t^C}{Y_t}$ is increasing in x_t.

Next, consider the (log) price of a $t+1$ period ahead nominal bond in period 0:

$$q_{0,t+1} = \ln \mathbb{E}_0 S_{0,t+1}$$

The effect on interest rates can be seen from the zeroth order expansion. We have

$$\begin{aligned}\bar{q}_{0,t+1} &= \ln\beta - \gamma\left(\bar{c}^C_{t+1} - \bar{c}^C_0\right) - \bar{x}_{t+1} \\ &= \ln\beta - \gamma\left[\ln\left(1-\exp\left(\bar{\phi}-\rho^{t+1}x_0\right)\right) - \ln\left(1-\exp\left(\bar{\phi}-x_0\right)\right)\right] - \rho^{t+1}x_0 \\ &= \ln\beta + \gamma\frac{\exp\left(\bar{\phi}\right)}{1-\exp\left(\bar{\phi}\right)}\left(1-\rho^{t+1}\right)x_0 - \rho^{t+1}x_0 + O\left(x_0^2\right). \end{aligned} \tag{9}$$

If ρ is sufficiently low or γ is sufficiently high, this expression is increasing in x_0, so expansionary money supply shock increases nominal price of bond or in other words decreases nominal interest rate. This is the standard mechanism in liqudity effect literature, such as Alvarez et al. (2001). It follows automatically that the real rate at all horizons is decreasing too. The simple version without capital already illustrates how the model is capable of overcoming the Canzoneri criticism (see Fact 2) that most conventional monetary models that rely on the aggregate demand channel suffer from.

Next, we move on to pricing risky claims. For simplicity consider a two-period lived asset whose terminal payoffs are co-linear with the aggregate non-wage income in period $t = 2$. The time $t = 0$ valuation of the claim is

$$\begin{aligned} V_0 &= \mathbb{E}_0 S_{0,2}[Y_t - (1-\lambda)N_t^W \frac{W_t}{P_t}] \\ &= \mathbb{E}_0 \left(\frac{C_2^C}{C_0^C}\right)^{-\gamma} \frac{1}{X_2} C_2^C. \end{aligned}$$

From zero-order terms, we have that both $\left(\frac{C_2^C}{C_0^C}\right)^{-\gamma}\frac{1}{X_2}$ and C_2^C increase in x_0, so these valuations are higher. A transitory monetary injection boosts the nominal income of the capitalists. So the non-wage share of total income goes up. This explains the increase in valuation from the cash flow channel.

Bernanke and Kuttner (2005) also emphasize the movements in equity premia. To see the effect on equity premia, we need to compute higher-order terms. We measure equity premium as expected returns on the period 2 non-wage income stream in excess of the two-period risk-free rate. Define returns as

$$R_2 \equiv \frac{Y_2 - (1-\lambda)N_2^W \frac{W_2}{P_2}}{V_0} \tag{10}$$

and equity premium as

$$\begin{aligned}\log EP_0 &\equiv \ln \frac{\mathbb{E}_0[Y_2 - (1-\lambda)N_2^W \frac{W_2}{P_2}]}{V_0} - \ln \frac{1}{Q_0^2} \\ &= \ln \mathbb{E}_0 C_2^C + \ln \mathbb{E}_0 S_{0,2} - \ln \mathbb{E}_0 S_{0,2} C_2^C .\end{aligned}$$

Using a second-order expansion, we obtain that

$$\begin{aligned}\log EP_0 &= \frac{\sigma^2}{2}\mathbb{E}_0 \left(\frac{dc_2^C}{d\sigma}\right)^2 \left[\left(1+\gamma^2\right) - (1-\gamma)^2\right] \\ &= \sigma^2 \gamma \frac{\exp\left(\bar{\phi} - \rho^2 x_0\right)}{1 - \exp\left(\bar{\phi} - \rho^2 x_0\right)} var\left(\epsilon\right) + \mathcal{O}(\sigma^3)\end{aligned}$$

We see that consistent with Bernanke and Kuttner (2005), a high x_0 lowers $\frac{\exp\left(\bar{\phi}-\rho^2 x_0\right)}{1-\exp\left(\bar{\phi}-\rho^2 x_0\right)}$ and therefore lowers equity premia.

Finally, we turn to the implications on the volatility and risk-adjusted volatility of log returns or the VIX. Using (10), we see that

$$V_0(\log R_2) = V_0(c_2^C)$$

and expression (8) says that the RHS is decreasing in x_0. The risk-adjusted vol is defined as

$$VIX_0 \equiv \mathbb{E}_0 S_{0,2} \left(\log R_2 - \mathbb{E} \log R_2\right)^2 \tag{11}$$

The next lemma shows conditions under which VIX declines too.

Lemma. *For all x_0,there exists (ρ, γ) such that $\partial_{x0} VIX_0 < 0 + \mathcal{O}(\sigma^3)$*

Proof. Rewrite equation (11) as

$$VIX_0 = \mathbb{E}_0 S_{0,2} V_0(\log R_2) + cov_0 \left\{S_{0,2}, \left(\log R_2 - \mathbb{E}\log R_2\right)^2\right\}$$

The covariance term is $\mathcal{O}(\sigma^3)$ and so to the second order,

$$\log VIX_0 = \log V_0(\log R_2) + q_{0,2}$$

To show that VIX goes down, we need to show that volatility falls more than the rise in the risk-free rate. From (8),

$$\partial_{x0} \log VOL = -\rho^2 - \frac{\exp\left(\bar{\phi} - \rho^2 x_0\right)\rho^2}{1 - \exp\left(\bar{\phi} - \rho^2 x_0\right)}$$

and from (9)

$$\partial_{x0} q_{0,2} = -\gamma \left[\frac{\exp\left(\bar{\phi} - \rho^2 x_0\right)\rho^2}{1 - \exp\left(\bar{\phi} - \rho^2 x_0\right)} \right] + \frac{\gamma \exp\left(\bar{\phi} - x_0\right)}{1 - \exp\left(\bar{\phi} - x_0\right)} - \rho^2$$

Thus for VIX to decline, we need

$$\frac{\gamma \exp\left(\bar{\phi} - x_0\right)}{1 - \exp\left(\bar{\phi} - x_0\right)} < (1+\gamma) \frac{\exp\left(\bar{\phi} - \rho^2 x_0\right)\rho^2}{1 - \exp\left(\bar{\phi} - \rho^2 x_0\right)} + 2\rho^2 \tag{12}$$

and for the interest rates to be decline we need

$$\frac{\gamma \exp\left(\bar{\phi} - x_0\right)}{1 - \exp\left(\bar{\phi} - x_0\right)} > \gamma \frac{\exp\left(\bar{\phi} - \rho^2 x_0\right)\rho^2}{1 - \exp\left(\bar{\phi} - \rho^2 x_0\right)} + \rho^2 \tag{13}$$

Notice that

$$\gamma \frac{\exp\left(\bar{\phi} - \rho^2 x_0\right)\rho^2}{1 - \exp\left(\bar{\phi} - \rho^2 x_0\right)} + \rho^2 < (1+\gamma) \frac{\exp\left(\bar{\phi} - \rho^2 x_0\right)\rho^2}{1 - \exp\left(\bar{\phi} - \rho^2 x_0\right)} + 2\rho^2$$

and hence there exists a non empty set of (ρ, γ) such that (12) and (13) are both satisfied.

□

The previous discussion shows that a model with segmented markets can generate patterns in aggregate income shares and asset prices that are consistent with the data. However, the analytically tractable case had constant output and strong effects on monetary injections on the price level. These shortcomings can be repaired by extending the analysis in several directions. The extensions retain the main channel that we highlighted so far – monetary policy injects liquidity to agents who consume out of non-wage income and price assets. We briefly mention the extensions and how the results are modified.

Investments It is straightforward to extend the model to allow for investment. Let the rental rate of capital be denoted by R_t^k. The optimality of the firms in the intermediate goods sector equate the marginal product of capital with the rental rate.

$$A\alpha \left(\frac{\lambda K_{t-1}}{1-\lambda} \right)^{\alpha-1} \left(\frac{\varepsilon_t - 1}{\varepsilon_t} \right) = R_t^k.$$

The capitalists now can save and smooth consumption via investing in capital. This gives us an Euler equation

$$U_c(C_t^T) = \beta \mathbb{E}_t U_c(C_{t+1}^C) [1-\delta] + \beta^2 \mathbb{E}_t U_c(C_{t+2}^C) R_{t+1}^k \left(\frac{P_{t+1}}{P_{t+2}} \right). \tag{14}$$

The equation relates the risk-adjusted marginal return on investment to the return on the risk-free bond. The next lemma show that a liquidity injection leads to higher capital stock in the future.

Lemma. *Around the steady-state, investment is increasing in money supply. In particular* $\partial_{x0} \left(\frac{I}{K} \right) > 0$.

To see the intuition for this result, consider what happens if the capitalist consumes the additional dollar in period in which the money supply went up. This will lower the marginal value of wealth today and keep the marginal value of future wealth as well as the marginal product of capital unchanged in the future periods. If the capitalist was optimizing in absence of the monetary shock, then the marginal value of future wealth times the return on his investment was equated to the marginal value of wealth today. Thus consuming the dollar entirely will will violate equation (14). The capitalist therefore saves a part of the increase in nominal wealth through capital. An increase in the liquidity of the capitalists (who are the investors) increases investment and output and aggregate consumption in future periods, producing the hump-shape pattern that is documented and emphasized in the empirical literature. At the same time, it reconciles the model with Canzoneri et al. (2007) findings, that in response to the shock expected consumption growth and expected path of nominal interest rates go in opposite directions.

Transfers, taxes Nominal transfers allow the government to satisfy its budget constraint through means other than seniorage. The easiest way to have lump-sum transfers for workers (or labor taxes). Adding such transfers would have several effects. First, nominal prices will react less to money injection since some of that being absorbed through taxes. This might be a desirable feature, since evidence summarized in Fact 1 suggest that prices react very little to monetary shocks. Second, it may also be a simple way to hit evidence on term premia (Fact 7) without having to introduce additional modeling implications.

Labor supply Proposition 1 relied on log utility for workers. Generalizing their preferences, we have

$$\begin{aligned} U_l\left(N_t\right)+\beta \mathbb{E}_t \frac{W_t}{P_{t+1}} U_c\left(C_{t+1}^W\right) &= 0 \\ N_t^{\varphi}+\mathbb{E}_t \frac{W_t}{P_{t+1} C_{t+1}^N}\left(C_{t+1}^W\right)^{1-\gamma} &= 0 \\ N_t^{\varphi}+\frac{1}{N_t} \mathbb{E}_t\left(C_{t+1}^W\right)^{1-\gamma} &= 0. \end{aligned}$$

A positive monetary shock lowers C_{t+1}^W. So N_t goes up in report to such shock if $\gamma > 1$.

Nominal frictions Adding nominal frictions will modify the equation $P_t = \frac{\varepsilon}{\varepsilon-1} W_t$ and replace it with

$$(1-\varepsilon)+\varepsilon \frac{W_t}{P_t}-\theta\left(\frac{P_t}{P_{t-1}}-1\right) \frac{P_t}{P_{t-1}}+\theta \beta \mathbb{E}_t \frac{U'\left(C_{t+1}^C\right)}{U'\left(C_t^C\right)}\left(\frac{P_{t+1}}{P_t}-1\right) \frac{P_{t+1}}{P_t} \frac{Y_{t+1}}{Y_t}=0,$$

and equation for C_t^C is more involved since W_t/P_t is no longer constant. Our analysis will work as follows. In response to a positive monetary shock (higher x_0), real wage W_t/P_t will increase. It will have some offsetting effect on capitalists, but one can make this effect arbitrarily small by lowering θ. So then qualitatively, the model will be consistent with higher real wages without changing anything more.

5 Calibrated Economy

In this section, we study a calibrated version of the setup laid out in section 3 and then use it to analyze the consequences of an unanticipated increase in the money supply.

We interpret the capitalists to be households that hold direct claims to risky capital. Following McKay and Reis (2013), we set the measure of capitalists to be 20%. The risk aversion of the capitalists and workers is set to 5 and 1, respectively. The discount factor is common and it set such that in the steady-state the risk-free rate is 2% annually. The capital share and depreciation rates are standard and set to 30% and 3%. The markup shocks are i.i.d, and calibrated such that the model obtains a average markup of 20% and a standard deviation of markups of 5%.

Parameter	Value
Subjective discount factor	0.98
Risk aversion of Capitalist, Workers	5,1
Frisch Elasticity for Workers	1
Capital share	1
Depreciation rate	0.03
Measure of Capitalists	0.2
Avg. Elasticity of substitution	10

Table 1: Calibration

We model the stochastic process for money growth and real transfers as

$$\frac{M_t}{M_{t-1}} = \exp\left\{\sigma_M \epsilon_{M,t}\right\}, \quad T_t = P_t \bar{T} \exp\left\{\sigma_T \epsilon_{M,t}\right\}.$$

where the common shock $\epsilon_{M,t}$ is i.i.d and the standard deviations σ_M and σ_T are set so that the model generates a standard deviation of the nominal rate as well as the inflation rate equal to 1% annually. The parameter choices are summarized in Table 1.

Our main exercise is to study an impulse response to a monetary expansion that lowers the nominal rate by 100 basis points in order to verify the predictions of our theory. The results are summarized in figure 1.

In the top panel of figure 1, we see that investment is higher, and output as increases by about 0.8%–increase in output results in a positive growth rate in consumption. Thus the model reconciles the VAR evidence (Fact 1) that documents that aggregate consumption peaks several quarters pursuing the monetary shock. With capital accumulation, the quantity theory equation does not hold exactly but is still a good guide to think about inflation. Given the monetary impulse is i.i.d and accomondated with an increase in lumpsum taxes, the inflation response is small. Most of the increase in aggregate output is due to return on capital being higher. The rightmost figure in the top panel plots the aggregate non-wage income, and we see that it increases and in our calibrated economy more than aggregate output.

The asset pricing results are in the bottom panel. An increase in the non-wage income implies that the capitalists see a lower expected growth of their consumption. This puts downward pressure on the real rate. An interesting outcome of this is that we obtain a negative co-movement of real rates and expected aggregate consumption growth thus resolving the Canzoneri et al. (2007) puzzle.

Finally, we turn to the equity valuations and returns. We define the value of the stock

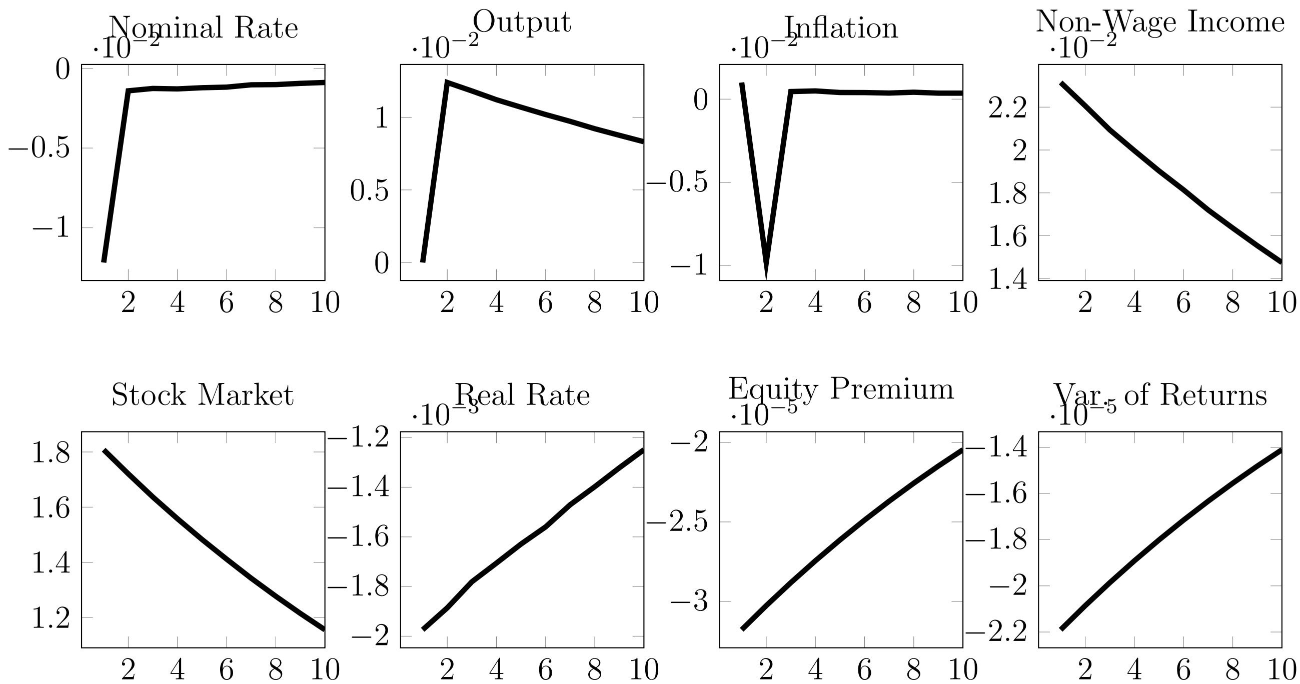

Figure 1: Impulse responses to monetary expansion

market as the present discount value of the claim to non-wage income minus investment. The model predicts an increase in the valuation of about 2%. Using the returns on this claim, we compute the equity premium and variance of returns and find that both are lower. In the current calibration, the quantitative magnitude of the fall in risk premium and volatility are smaller than the evidence cited in Facts 5 -6.

6 Conclusion

This paper presented a model with segmented market that is consistent with the empirical literature on how central bank actions affect the macroeconomy and financial markets.

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